4 | Quadratic Equations

Quadratic Polynomial

A polynomial of the form $ax^2 + bx + c$, $a \neq 0$ is called a Quadratic polynomial, Degree of this polynomla is 2

Quadratic Equation

An equation of the form $ax^2 + bx + c = 0$, $a \neq 0$ is called a Quadratic equation, in one variable x, where a, b, c are real numbers.

Roots of a Quadratic Equations

A real number α is called root of $ax^2 + bx + c = 0$, when it satisfies this equation i.e., $a\alpha^2 + b\alpha + c = 0$. A quadratic equation have at most 2 real roots.

Methods for solving Quadratic Equations

By factorisation

By Using Quadratic Formula (Given by Sridhar Acharya)

(a) By using identities
(b) By splitting the middle term

Quadratic equation $ax^2 + bx + c = 0$ has two roots α and β given by

$$\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \; , \; \beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Discriminant
$D = b^2 - 4ac$ is called discriminant

Nature of roots

Case I : $D > 0$, $b^2 - 4ac > 0$
Roots are real and distinct

Case II : $D = 0$, $b^2 - 4ac = 0$
Roots are real and equal

Case III : $D < 0$, $b^2 - 4ac < 0$
Roots are not real

Real roots can be rational or irrational
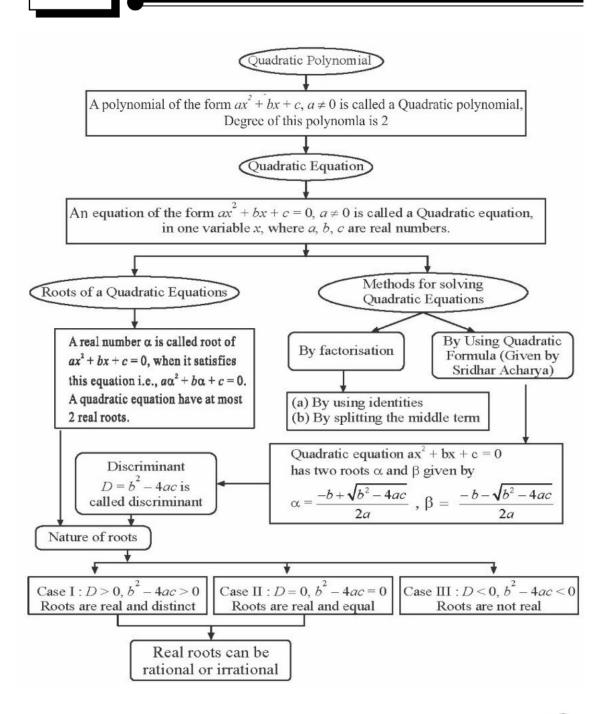
Mathematics-X

NOTES:

1. Real and distinct roots are $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

2. Real and equal roots are $\dfrac{-b}{2a}, \dfrac{-b}{2a}$

3. There are quadratic equation which donot have any real roots e.g. $x^2 + 1 = 0$

VERY SHORT ANSWER TYPE QUESTIONS

Multiple Choice Questions:

1. Which of the following is not a Quadratic Equation?
   (a) $2(x-1)^2 = 4x^2 - 2x + 1$
   (b) $3x - x^2 = x^2 + 6$
   (c) $\left(\sqrt{3}x + \sqrt{2}\right)^2 = 2x^2 - 5x$
   (d) $(x^2 + 2x)^2 = x^4 + 3 + 4x^2$

2. Which of the following equation has 2 as a root
   (a) $x^2 + 4 = 0$
   (b) $x^2 - 4 = 0$
   (c) $x^2 + 3x - 12 = 0$
   (d) $3x^2 - 6x - 2 = 0$

3. If $\dfrac{1}{2}$ is a root of $x^2 + px - \dfrac{5}{4} = 0$ then value of p is
   (a) 2
   (b) –2
   (c) $\dfrac{1}{4}$
   (d) $\dfrac{1}{2}$

4. Every Quadratic Equation can have at most
   (a) Three roots
   (b) One root
   (c) Two roots
   (d) Any number of roots

5. Roots of Quadratic equation $x^2 - 7x = 0$ will be
   (a) 7
   (b) 0, –7
   (c) 0, 5
   (d) 0, 7

6. The value(s) of k for which the quadratic equation $2x^2 + kx + 2 = 0$ has equal roots, is
   (a) 4
   (b) ± 4
   (c) -4
   (d) 0
   (CBSE 2020)

7. **Fill in the blanks:**

   (a) If $px^2 + qx + r = 0$ has equal roots then value of r will be _____ .

   (b) The quadratic equation $x^2 - 5x - 6 = 0$ if expressed as $(x + p)(x + q) = 0$ then value of p and q respectively are _____ and _____ .

   (c) The value of k for which the roots of qaudratic equations $x^2 + 4x + k = 0$ are real is _____ .

   (d) If roots of $4x^2 - 2x + c = 0$ are reciprocal of each other then the value of c is _____ .

   (e) If in a quadratic equation $ax^2 + bx + c = 0$, value of a is zero then it become a _____ equation.

8. **Write the discriminant of the quadractic equation $(x+5)^2 = 2(5x-3)$**

9. Roots of $-x^2 + \dfrac{1}{2}x + \dfrac{1}{2} = 0$

   (a) $-\dfrac{1}{2}, 1$ (b) $\dfrac{1}{2}, 1$

   (c) $\dfrac{-1}{2}, -1$ (d) $\dfrac{1}{2}, \dfrac{-1}{2}$

SHORT ANSWER TYPE QUESTIONS-I

10. If the quadratic equation $px^2 - 2\sqrt{5}\,px + 15 = 0$ $(p \neq 0)$ has two equal roots then find the value of p.

11. **Solve for x by factorisation**

    (a) $8x^2 - 22x - 21 = 0$

    (b) $3\sqrt{5}x^2 + 25x + 10\sqrt{5} = 0$

    (c) $2x^2 + ax - a^2 = 0$ **(CBSE 2014)**

    (d) $3x^2 - 2\sqrt{6}x + 2 = 0$ **(CBSE 2010)**

    (e) $\sqrt{3}x^2 + 10x + 7\sqrt{3} = 0$

    (f) $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

    (g) $(x - 1)^2 - 5(x - 1) - 6 = 0$

Mathematics-X

12. For what vlaue of 'a' quadratic equation $3ax^2 - 6x + 1 = 0$ has no real roots?

(CBSE 2020)

13. If -5 is a root of the quadratic equation $2x^2 + px - 15 = 0$ and the quadratic equation $p(x^2 + x) + k = 0$ has equal roots find the value of k.

(CBSE 2014, 2016)

14. If $x = \dfrac{2}{3}$ and $x = -3$ are roots of the quadratic equation $ax^2 + 7x + b = 0$. Find the value of a and b.

(CBSE 2016)

15. Find value of p for which the product of roots of the quadratic equation $px^2 + 6x + 4p = 0$ is equal to the sum of the roots.

16. The sides of two squares are x cm and $(x + 4)$ cm. The sum of their areas is 656 cm^2 Find the sides of these two squares.

17. Find k if the difference of roots of the quadratic equation $x^2 - 5x + (3k - 3) = 0$ is 11.

SHORT ANSWER TYPE QUESTIONS-II

18. Find the positive value of k for which the quadratic equation $x^2 + kx + 64 = 0$ and the quadratic equation $x^2 - 8x + k = 0$ both will have real roots.

19. Solve for x

(a) $\dfrac{1}{a+b+x} = \dfrac{1}{a} + \dfrac{1}{b} + \dfrac{1}{x}$ $a + b + x \neq 0,$ **(CBSE 2005)**

$a, b, x \neq 0$

(b) $\dfrac{1}{2a+b+2x} = \dfrac{1}{2a} + \dfrac{1}{b} + \dfrac{1}{2x}$ $2a + b + 2x \neq 0,$

$a, b, x \neq 0$

(c) $\dfrac{2x}{x-3} + \dfrac{1}{2x+3} + \dfrac{3x+9}{(x-3)(2x+3)} = 0, x \neq 3, \dfrac{-3}{2}$

(d) $4x^2 + 4bx - (a^2 - b^2) = 0$

(e) $\dfrac{1}{x-1} - \dfrac{1}{x+5} = \dfrac{6}{7}, x \neq 1, 5$ **(CBSE 2010)**

(f) $4x^2 - 2(a^2 + b^2)x + a^2b^2 = 0$

(g) $\dfrac{2}{x+1} + \dfrac{3}{2(x-2)} = \dfrac{23}{5x}, x \neq 0, -1, 2$

(h) $\left(\dfrac{2x}{x-5}\right)^2 + \dfrac{10x}{(x-5)} - 24 = 0, x \neq 5$

(i) $4x^2 - 4a^2x + a^4 - b^4 = 0$

(j) $2a^2x^2 + b(6a^2 + 1)x + 3b^2 = 0$

(k) $3\left(\dfrac{7x+1}{5x-3}\right) - 4\left(\dfrac{5x-3}{7x+1}\right) = 11, \ x \neq \dfrac{3}{5}, \ \dfrac{-1}{7}$

(l) $\dfrac{1}{x+4} - \dfrac{1}{x-7} = \dfrac{11}{30}, \ x \neq -4, 7$

(m) $\dfrac{x-4}{x-5} + \dfrac{x-6}{x-7} = \dfrac{10}{3}, \ x \neq 5, 7$ **(CBSE 2014)**

(n) $\dfrac{1}{x+1} + \dfrac{2}{x+2} = \dfrac{4}{x+4}, \qquad x \neq -1, -2, -4$

(o) $\dfrac{1}{2x-3} + \dfrac{1}{x-5} = 1, \qquad x \neq \dfrac{3}{2}, 5$

(p) $x^2 + 5\sqrt{5}x - 70 = 0$

(q) $\dfrac{16}{x} - 1 = \dfrac{15}{x+1}, \ x \neq 0, -1$ **(CBSE 2014)**

20. Solve by using quadratic formula $abx^2 + (b^2 - ac)\,x - bc = 0$. **(CBSE 2005)**

21. If the roots of the quadratic equation $(p + 1)x^2 - 6(p + 1)\,x + 3(p + 9) = 0$ are equal find p and then find the roots of this quadratic equation.

22. Find the nature of roots of the quadratic equation $3x^2 - 4\sqrt{3}\,x + 4 = 0$

    If the roots are real, find them. **(CBSE 2020)**

23. Solve $9x^2 - 6a^2x + a^4 - b^4 = 0$ using quadratic formula. **(CBSE 2020)**

LONG ANSWER TYPE QUESTIONS

24. A train travels at a certain average speed for a distance of 54 km and then travels a distance of 63 km at an average speed of 6 km/hr more than the first speed. If it takes 3 hours to complete the total journey, what is its first speed?

25. A natural number, when increased by 12, equals 160 times its reciprocal. Find the number.

26. A thief runs with a uniform speed of 100 m/minute. After one minute a policeman runs after the thief to catch him. He goes with a speed of 100 m/minute in the first minute and increases his speed by 10 m/minute every succeeding minute. After how many minutes the policemen will catch the thief?

27. Two water taps together can fill a tank in 6 hours. The tap of larger diameter takes 9 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank. **(CBSE 2020)**

28. In the centre of a rectangular lawn of dimensions 50 m × 40 m, a rectangular pond has to be constructed, so that the area of the grass surrounding the pond would be 1184 m². Find the length and breadth of the pond.

29. A farmer wishes to grow a 100 m² recangular garden. Since he has only 30 m barbed wire, he fences three sides of the rectangular garden letting compound wall of this house act as the fourth side fence. Find the dimensions of his garden.

30. A peacock is sitting on the top of a pillar, which is 9 m high. From a point 27 m away from the bottom of a pillar, a snake is coming to its hole at the base of the pillar. Seeing the snake the peacock pounces on it. If their speeds are equal at what distance from the hole is the snake caught?

31. If the price of a book is reduced by ₹ 5, a person can buy 5 more books for ₹ 300. Find the original list price of the book.

32. ₹ 6500 were divided equally among a certain number of persons. If there been 15 more persons, each would have got ₹ 30 less. Find the original number of persons.

33. In a flight of 600 km, an aircraft was slowed down due to bad weather. Its average speed was reduced by 200 km/hr and the time of flight increased by 30 minutes. Find the duration of flight. **(CBSE 2020)**

34. A fast train takes 3 hours less than a slow train for a journey of 600 km. If the speed of the slow train is 10 km/hr less than the fast train, find the speed of the two trains.

(CBSE 2020)

35. The speed of a boat in still water is 15 km/hr. It can go 30 km upstream and return downstream to the orignal point in 4 hrs 30 minutes. Find the speed of the stream.

36. Sum of areas of two squares is 400 cm². If the difference of their perimeter is 16 cm. Find the side of each square.

37. The area of an isosceles triangle is 60 cm². The length of equal sides is 13 cm find length of its base.

38. The denominator of a fraction is one more than twice the numerator. If the sum of the fraction and its reciprocal is $2\dfrac{16}{21}$. Find the fraction.

39. A girl is twice as old as her sister. Four years hence, the product of their ages (in years) will be 160. Find their present ages.

40. A two digit number is such that the product of its digits is 18. When 63 is subtracted from the number, the digits interchange their places. Find the number.

(CBSE 2006)

41. Three consecutive positive integers are such that the sum of the square of the first and the product of other two is 46, find the integers. **(CBSE 2010)**

42. A piece of cloth costs ₹ 200. If the piece was 5 m longer and each metre of cloth costs ₹ 2 less, then the cost of the piece would have remained unchanged. How long is the piece and what is the original rate per metre?

43. A motor boat whose speed is 24 km/hr in still water takes 1 hour more to go 32 km upstream than to return downstream to the same spot. Find the speed of the stream

(CBSE 2016)

44. If the roots of the quadratic equation $(b - c)x^2 + (c - a)x + (a - b) = 0$ are equal, Prove that $2b = a + c$.

45. If the equation $(1 + m^2)n^2x^2 + 2mncx + (c^2 - a^2) = 0$ has equal roots, prove that $c^2 = a^2 (1 + m^2)$.

46. A train covers a distance of 480 km at a uniform speed. If the speed had been 8 km/hr less, then it would have taken 3 hours more to cover the same distance. Find the original speed of the train. **(CBSE 2020)**

47. A rectangular park is to be designed whose breadth is 3 m less than its length. Its area is to be 4 square metres more than the area of a park that has already been made in the shape of an isosceles triangle with its base as the breadth of the rectangular park and of altitude 12 m. Find the length and breadth of the park.

(CBSE 2020)